Extrapolate: generalizing counterexamples of functional test properties

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ABSTRACT
This paper presents a new tool called Extrapolate that automatically generalizes counterexamples found by property-based testing in Haskell. Example applications show that generalized counterexamples can inform the programmer more fully and more immediately what characterises failures. Extrapolate is able to produce more general results than similar tools. Although it is intrinsically un-sound, as reported generalizations are based on testing, it works well for examples drawn from previous published work in this area.

CCS CONCEPTS
• Software and its engineering → Software testing and debugging;

KEYWORDS
enumerative property-based testing, systematic testing, functional programming, Haskell.

ACM Reference Format:

1 INTRODUCTION
Most programmers are familiar with the following situation: a failing test case has been discovered during testing; but it is not immediately apparent what more general class of tests would trigger the same failure. The programmer may resort to painstaking step-by-step reevaluation of the reported failure in the hope of realizing where a fault lies. In this paper, we examine a less explored approach: the generalization of failing cases informs the programmer more fully and more immediately what characterises such failures. This information helps the programmer to locate more confidently and more rapidly the causes of failure in their program. We present Extrapolate, a tool to generalize counterexamples of test properties in Haskell. Several example applications demonstrate the effectiveness of Extrapolate.

Example 1.1. Consider the following faulty sort function:

```haskell
sort :: Ord a => [a] -> [a]
sort [] = []
sort (x:xs) = sort (filter (< x) xs) ++ [x] ++ sort (filter (> x) xs)
```

The function `sort` should have the following properties:

```haskell
prop_sortOrdered :: Ord a => [a] -> Bool
prop_sortOrdered xs = ordered (sort xs)

prop_sortCount :: Ord a => a -> [a] -> Bool
prop_sortCount x xs = count x (sort xs) == count x xs
```

That together completely specify `sort`.

If we pass both properties to an established property-testing library, such as QuickCheck [11] or SmallCheck [33], we get something like:

```haskell
> check (prop_sortOrdered :: [Int] -> Bool)
+++ OK, passed 500 tests.

> check (prop_sortCount :: Int -> [Int] -> Bool)
*** Failed! Falsifiable (after 4 tests):
0 [0,0]
```

That is, `prop_sortCount 0 [0,0] = False`. If instead we test using Extrapolate, then for the failing property, in addition to a specific counterexample, Extrapolate prints:

```haskell
Generalization:
  x (x:x:_)
```

Some values have been generalized: the specific value 0 does not matter, `prop_sortCount x (x:x:_)` is false for any integer `x`; the tail value _ does not affect the result. Extrapolate also prints:

```haskell
Conditional Generalization:
  x (x:xs) when elem x xs
```

This hints that our faulty sort function fails for lists with repeated elements. We return to this example in §4.1.

In general, if Extrapolate finds a counterexample, then the property is definitely false. Any specific counterexample displayed by Extrapolate is valid. A displayed generalization may or may not be valid. However, a generalization is only displayed if a specific counterexample has first been found and displayed. So Extrapolate never incorrectly reports a property to be false, even though a generalized counterexample may be invalid (or `loose`, cf. §6).
1.1 Contributions
The main contributions of this paper are:

1. methods using automated black-box testing to generalize counterexamples of functional test properties by replacing constructors with variables, where these variables may be repeated or subject to side-conditions;
2. the design of the Extrapolate library, which implements these methods in Haskell and for Haskell test properties;
3. a selection of small case-studies, investigating the effectiveness of Extrapolate;
4. a comparative evaluation of generalizations performed by Extrapolate and similar tools for Haskell.

Despite the Haskell setting of the implementation and experiments, we expect similar techniques to be applicable in other functional programming languages.

1.2 Road-map
This paper is organized as follows:

§2 describes how to use Extrapolate;
§3 describes how Extrapolate works internally;
§4 presents example applications and results;
§5 discusses related work;
§6 evaluates Extrapolate in comparison with similar tools;
§7 draws conclusions and suggests future work.

2 HOW EXTRAPOLATE IS USED
Extrapolate is a library (loaded by “import Test.Extrapolate”). Unless they already exist, instances of the Listable (§3.1) and Generalizable (§3.2) typeclasses are declared for needed user-defined datatypes (step 1). The check function is then applied to each test property (step 2).

Step 1. Provide class instances for used-defined types. Extrapolate needs to know how to generate test values for property arguments — this capability is provided by instances of the Listable typeclass (§3.1). Extrapolate also needs to manipulate the structure of test values so that it can perform its generalization procedure — this capability is provided by instances of the Generalizable typeclass (§3.2). Extrapolate provides instances for most standard Haskell types and a facility to derive instances for user-defined data types using Template Haskell [34]. For applications in which values of algebraic datatypes can be freely constructed, as there are no constraining data invariants, writing deriveGeneralizable "<Type> is enough to create the necessary instances. Extrapolate’s online documentation explains how to define these instances manually.

Step 2. Call the property-checking function. The function check tests properties. If counterexamples are found, it reports them and any candidate generalizations.

Example 1.1 (revisited). Figure 1 shows the program used to obtain the results in §1.

When exploring conditional generalizations, Extrapolate limits conditions by size. Size is defined as the number of symbols in an expression added to the sizes of constants as defined by LeanCheck’s

import Test.Extrapolate
sort :: Ord a => [a] -> [a]
sort [] = []
sort (x:xs) = sort (filter (< x) xs) ++ [x] ++ sort (filter (> x) xs)

prop_sortOrdered :: [Int] -> Bool
prop_sortOrdered xs = ordered (sort xs)

where
ordered :: [Int] -> Bool
ordered (x:y:xs) = x <= y && ordered (y:xs)
ordered _ = True

prop_sortCount :: Int -> [Int] -> Bool
prop_sortCount x xs = count x (sort xs) == count x xs

where
count :: Int -> [Int] -> Int
count x = length . filter (== x)

main :: IO ()
main = do
  check prop_sortOrdered
  check prop_sortCount

Figure 1: Full program applying Extrapolate to properties of sort used to obtain the results in §1.

Listable enumeration (§3.1). This definition is similar to the one used in our previous work on Speculate [5]. By default, Extrapolate:

- tests properties for up to 500 value assignments;
- considers side conditions up to size 4;
- uses Speculate to find equivalences between expressions of up to size 4 avoiding many redundant equivalent conditions (§3.3).

Extrapolate allows variations of these default settings. The number of configured tests affects the runtime linearly so long as the underlying Listable enumeration has linear runtime. The size limit of side conditions affects runtime exponentially so it should be adjusted with caution. Our choice of default parameters aims for a runtime of a few seconds. For all the examples in §4 Extrapolate terminates in less than 10 seconds.

3 HOW EXTRAPOLATE WORKS
Extrapolate works in three steps:

1. it tests properties searching for counterexamples, and if any is found, steps 2 and 3 are performed (§3.1);
2. it tries to generalize counterexamples by substituting variables for constants (§3.2);
3. it tries to generalize counterexamples by introducing variables subject to side conditions (§3.3).
Extrapolate: generalizing counterexamples
of functional test properties

Throughout this section we shall use the following example to illustrate how each step of Extrapolate works.

Example 3.1. In Haskell, the Data.List module declares the function nub, that removes duplicate elements from a list, keeping only their first occurrences. Consider the following incorrect property about nub:

\[
\text{prop_nubid :: [Int] -> Bool}
\]

\[
\text{prop_nubid \ x s = \ nub \ x s = \ x s}
\]

When Extrapolate’s check function is applied to the above property, it produces the following output:

> check prop_nubid

*** Failed! Falsifiable (after 3 tests): 
\[
[0,0]
\]

Generalization:

\[
x:x:_
\]

Conditional Generalization:

\[
x:x \ \text{when elem} \ x \ x
\]

Coincidentally, the result for this example is similar to the result we saw in §1 — but it is simpler.

\[\Box\]

3.1 Searching for counterexamples

To test properties searching for counterexamples, Extrapolate uses LeanCheck [2] which defines the Listable typeclass used to generate test values enumeratively.

We have used LeanCheck previously in other tools including Speculate [5], a tool we use when generating conditional generalizations (§3.3). More information on LeanCheck and the Listable typeclass can be found in its Haddock documentation [3] and in §4.1 and §4.2 of [4].

Example 3.1 (1st revisit). When Extrapolate’s check function is applied to prop_nubid, it is tested for each of the list arguments: [], [0], [0,0]. The property fails for the third list [0,0] and Extrapolate produces its first two lines of output:

*** Failed! Falsifiable (after 3 tests):

\[
[0,0]
\]

\[\Box\]

3.2 Unconditional generalization

Listing generalizations. After finding a counterexample, Extrapolate lazily lists all of its possible generalizations, from most general to least general, formed by replacing one or more subexpressions with variables. Generality order is not total as some generalizations are incomparable. So we consider candidate replacements left-to-right.

Example 3.1 (2nd revisit). Recall the reported counterexample to prop_nubid: [0,0], or, more verbosely, 0:0:[]. Its generalizations from most general to least general are: xs, x:xs, x:y:xs, x:x:xs, x:y:[]; x:x:[]; x:0:xs, x:0:[]; 0:xs, 0:x:xs, 0:x:[] and 0:0:xs.

\[\Box\]

3.3 Conditional generalization

Background functions. Before trying to discover conditional generalizations we must first decide which background functions are allowed to appear in conditions.

The larger the number of functions in the background, the longer Extrapolate will take to produce a conditional generalization. So, we refrain from including a large set such as the entire Haskell Prelude. If the algorithm used is ever refined to something faster we may be able to include a larger set by default (§7).

Each type has a default list of functions to be included in the background. These lists are declared as part of Generalizable typeclass instances:

- for Ints, ==, /=, <=, <;
- for Chars, ==, /=, <=, <;
- for Bool, ==, /=, not;
- for lists, ==, /=, <=, <, length and elem;
- for Maybe, ==, /=, <=, <, Just;
- for tuples, ==, /=, <=, <.

For user-defined datatypes, the implementor of instances of the Generalizable typeclass decides which functions to include in the background. When using deriveGeneralizable, by default, Eq instances have == and /= in the background and Ord instances have < and <= in the background. Additional background functions can be provided using the ‘withBackground’ combinator. In our experiments (§4), we found it useful to include in the background functions already appearing in properties.

Whenever a property is tested, these background functions are gathered for all types and component types of arguments of the property being tested. The background of Bool is always included.

Background constants. Constants of the types being tested, obtained from their Listable instances, are also included in the background.
Example 3.1 (4th revisit). The background functions used when testing prop_nubid :: [Int] -> Bool are

\[
\begin{align*}
(=) & : \text{Bool} \rightarrow \text{Bool} \\
(/=) & : \text{Bool} \rightarrow \text{Bool} \\
(\leq) & : \text{Int} \rightarrow \text{Int} \rightarrow \text{Bool} \\
(<) & : \text{Int} \rightarrow \text{Int} \rightarrow \text{Bool} \\
\text{not} & : \text{Bool} \rightarrow \text{Bool} \\
\text{length} & : \text{Int} \rightarrow \text{Int} \\
\text{elem} & : \text{Int} \rightarrow \text{Int} \rightarrow \text{Bool}
\end{align*}
\]

along with enumerated constants of \text{Bool}, \text{Int} and \text{[Int]} types. □

Enumerating expressions. Extrapolate uses Speculate [5] to recursively enumerate expressions formed by type-correct applications of Boolean functions occurring in properties and variables. Speculate avoids generating many distinct but semantically equivalent expressions by using results of previous tests to determine equivalence classes of subexpressions, and term-rewriting to normalize subexpressions to canonical forms. For further details see [5]. Expressions are enumerated lazily and limited by the configured maximum size. This enumeration process is done only once for each property that has a counterexample.

Enumerating candidate conditions. Expressions of Boolean type are used as candidate side-conditions.

Example 3.1 (5th revisit). With Extrapolate configured to consider conditions up to size 3, Speculate returns the following 24 conditions

\[
\begin{align*}
(1) \ p & \quad (9) \ \text{x} \leq \text{ys} & (17) \ \text{x} /= \text{y} \\
(2) \ \text{False} & \quad (10) \ \text{x} < \text{ys} & (18) \ \text{x} /= \text{0} \\
(3) \ \text{True} & \quad (11) \ \text{elem} \ \text{x} \ \text{xs} & (19) \ \text{x} < \text{y} \\
(4) \ \text{not} \ \text{p} & \quad (12) \ \text{elem} \ 0 \ \text{xs} & (20) \ \text{x} < \text{0} \\
(5) \ \text{xs} \ = \ \text{ys} & \quad (13) \ \text{p} = \text{q} & (21) \ 0 < \text{x} \\
(6) \ \text{xs} \ = \ [] & \quad (14) \ \text{p} /= \text{q} & (22) \ \text{x} <= \text{y} \\
(7) \ \text{x} /= \ [] & \quad (15) \ \text{x} = \text{y} & (23) \ \text{x} < \text{0} \\
(8) \ \text{x} /= \ [] & \quad (16) \ \text{x} = \text{0} & (24) \ 0 <= \text{x}
\end{align*}
\]

where \text{p} :: \text{Bool}; \text{x,y} :: \text{Int}; \text{xs,ys} :: \text{[Int]}. In addition, repeated variable instances of these conditions are tested when searching for candidate conditional generalizations. □

Discarding neutral side-conditions. Neutral conditions are those yielding generalizations equivalent to a simpler counterexample. For example \([x,x]\) when \text{x} == 0 is equivalent to simply \([0,0]\). So any condition of the form \langle \text{var} \rangle = \langle \text{value} \rangle is discarded. So too is any condition containing at least one variable \text{v} and found true only for a single assignment of a value to \text{v}.

Turning off Speculate. The use of Speculate is optional and can be turned off by: check ‘maxSpeculateSize’ 0. Using Speculate has two effects in most cases:

1. runtime is significantly reduced;
2. detection of neutral conditions is improved.

Listing conditional generalizations. For each generalization, Extrapolate produces all possible Boolean conditions involving its variables based on the list of candidate conditions — this includes all possible variable renamings.

Testing conditional generalizations. Extrapolate tests to find side conditions under which the property fails all tests. Of these conditions, Extrapolate selects the conditions satisfied in the most test cases. Remember different side conditions may hold for distinct numbers of test values.

This is similar to what we do in Speculate [5]: there we find side-conditions to properties, whereas here we apply side-conditions to generalized counterexamples.

Example 3.1 (6th revisit). Suppose Extrapolate has been configured to explore conditions of up to size 3. For the first candidate generalization, replacing \([0,0]\) by \text{xs}, two candidate conditions are listed:

\[
\begin{align*}
(1) \ \text{xs} /= [] & \\
(2) \ \text{elem} \ 0 \ \text{xs}
\end{align*}
\]

Testing shows that the candidate generalization \text{xs} does not always falsify the property under either of these conditions. For example, the property does not fail for the list \([0]\).

For the second candidate generalization, replacing \(\emptyset:0:[]\) by \text{x:xs}, Extrapolate lazily lists eight candidate conditions:

\[
\begin{align*}
(1) \ \text{x} /= [] & \\
(2) \ \text{elem} \ 0 \ \text{xs} & (3) \ \text{elem} \ 0 \ \text{xs} & (6) \ 0 < \text{x}
\end{align*}
\]

The property only fails for all test values of the form \text{x:xs} under the second condition, \text{elem x xs}. Extrapolate reports its last two lines of output:

\[
\text{Conditional Generalization:} \quad \text{x:xs when elem x xs}
\]

4 Example Applications and Results

In this section, we use Extrapolate to generalize counterexamples of properties about:

- a sorting function (§4.1);
- a calculator library (§4.2);
- a serializer and parser (§4.3);
- the XMonad window manager (§4.4).

These example applications are adapted from Pike [30]. We omit just two of Pike’s examples: one concerns arithmetic overflow and for the other we have incomplete information. The example applications in §4.2, §4.3 and §4.4 are respectively of small, medium and large scale. In §4.5, we use generalizations as property refinements. In §4.6 we give a summary of performance results for all these applications.

It was during our experiments with these examples that we found a simple rule of thumb for improving the results of Extrapolate: to include in the background functions occurring in properties. Automatically including these functions is left as future work (§7).

The complete source code for all examples described here is included in the Extrapolate distribution package. The package includes other examples not discussed here for the sake of space.

In this section, we evaluate Extrapolate on its own. Comparison with related work can be found in §6.
4.1 A sorting function: exact generalization
Carrying on from the example described in §1 and Figure 1, if we include count as a background function (§3.3),

```haskell
> let chk = check 'withConditionSize' 6
>           'withBackground' [constant "count" count]
> chk prop_sortCount
```

Extrapolate prints:
Conditional Generalization:
x xs when count x xs > 1
This is an exact description of the test cases that fail.

4.2 A calculator language
In this section, we use Extrapolate to find and generalize counterexamples to a property about the simple calculator language described by Pike [30].

Expressions to be calculated are represented by the datatype `Exp` and may contain integer constants, addition and division.

```haskell
data Exp = C Int
         | Add Exp Exp
         | Div Exp Exp
```

The function `eval` evaluates `Exp`s and returns a `Maybe` value:
- Nothing when the calculation involves a division by 0;
- Just an integer otherwise.

```haskell
eval :: Exp -> Maybe Int
eval (C i) = Just i
eval (Add e0 e1) = liftM2 (+) (eval e0) (eval e1)
eval (Div e0 e1) =
    let e = eval e1
    in if e == Just 0
        then Nothing
        else liftM2 div (eval e0) e
```

The following function `noDiv0`, returns True when no literal division by 0 occurs in an expression.

```haskell
noDiv0 :: Exp -> Bool
noDiv0 (C _) = True
noDiv0 (Div _ (C 0)) = False
noDiv0 (Add e0 e1) = noDiv0 e0 && noDiv0 e1
noDiv0 (Div e0 e1) = noDiv0 e0 && noDiv0 e1
```

Using noDiv0, we define the following test property:

```haskell
\e -> noDiv0 e ==> eval e /= Nothing
```

That is, if an expression contains no literal division by 0, evaluating it returns a Just value.

Using Extrapolate, we find a counterexample and two generalizations:

```haskell
> check $ \e -> noDiv0 e ==> eval e /= Nothing
*** Failed! Falsifiable (after 20 tests):
Div (C 0) (Add (C 0) (C 0))
```

4.3 A serializer and parser
In this section, we apply Extrapolate to the parser and pretty printer for a toy language described by Pike [30].

Programs in this language are represented by the datatype `Prog`, which can contain modules, functions, statements, expressions, assignments, etc. For brevity, we omit details of the implementation here, but it is included in the Extrapolate distribution package. It has two main functions:

```haskell
show' :: Prog -> String
read' :: String -> Prog
```

The serializer (`show'`) is defined in ≈100 lines of code. The parser (`read'`) is defined in ≈200 lines of code. The parser includes a bug that switches the arguments of conjunctions.

When we test the property that serializing followed by parsing is an identity, Extrapolate reports a counterexample along with generalizations:

```haskell
> check $ \e -> read' (show' e) == e
*** Failed! Falsifiable (after 96 tests):
Prog [] [Func (Var "a")
    [And (Int 0) (Bool False)]]
```

Generalization:

```haskell
Div (C _) (Add (C 0) (C 0))
```

Conditional Generalization:

```haskell
Div e1 (Add (C 0) (C 0)) when noDiv0 e1
```

The property fails because it is not enough to test whether any denominator is a literal zero constant, we should test whether any denominator evaluates to zero. The generalized counterexamples provide improved information for the programmer. Specifically, constructors unrelated to the fault are generalized to variables.

To generate the above conditional generalization we manually included noDiv0 in the list of background functions.

The following maximal generalizations (§6) are out-of-reach for the current implementation of Extrapolate as the conditions are too large with 9 and 16 symbols respectively.

```
Div e1 e2 when noDiv0 (Div e1 e2) && eval e2 == Just 0
Div e1 e2 when noDiv0 e1 && noDiv0 e2 && e2 /= (C 0) && eval e2 == Just 0
```

At least one reader has suggested smaller expressions for these conditions. They were mistaken! Machine checking is useful!
4.4 XMonad

XMonad [36] is a tiling window manager written in roughly 1700 lines of Haskell code. The XMonad developers defined over a hundred test properties.

In this section, we use Extrapolate to find an artificial bug introduced by Pike [30] in XMonad.

The function. XMonad has a function

\[
\text{removeFromWorkspace ws = ws \{ stack = stack ws \gg= filter (\neq w) \}}
\]

which removes a given window from a given workspace.

The bug. As described by Pike [30], we introduce an artificial bug, replacing \( \neq \) by \( = \) simulating a typo:

\[
\text{ws \{ stack = stack ws \gg= filter (==w) \}}
\]

The property. The following property \prop_delete\ is one of XMonad’s original test properties.

\[
\text{prop_delete x = case peek x of}
\]

\[
\text{Nothing -> True}
\]

\[
\text{Just i -> not (member i (delete i x))}
\]

A counterexample. In a regular enumerative property-based testing tool, we would get the following minimal counterexample for \prop_delete\ (indented to improve readability):

\[
> \text{check prop_delete}
\]

Failed! Falsifiable (after 15 tests):

\[
\text{StackSet \{} \text{current = Screen}
\]

\[
\text{\{} \text{workspace = Workspace}
\]

\[
\text{\{} \text{tag = 0}
\]

\[
\text{\{} \text{layout = 0}
\]

\[
\text{\{} \text{stack = Just (Stack \{} \text{focus = 'a'}
\]

\[
\text{\{} \text{up = ''}
\]

\[
\text{\{} \text{down = ''}
\]

\[
\text{\}} \})
\]

\[
, \text{screen = 0}
\]

\[
, \text{screenDetail = 0 }
\]

\[
, \text{visible = [ ]}
\]

\[
, \text{hidden = [ ]}
\]

\[
, \text{floating = fromList [ ] }
\]

A generalization. When we pass \prop_delete\ to Extrapolate, we instead get the following output (again indented to improve readability):

\[
> \text{check prop_delete}
\]

*** Failed! Falsifiable (after 15 tests):

\[
\text{StackSet (Screen (Workspace 0 0 (Just (Stack \{} \text{focus = 'a'}}
\]

\[
\text{\{} \text{up = ''}
\]

\[
\text{\{} \text{down = ''}
\]

\[
\text{\}} \})
\]

\[
, \text{screen = 0}
\]

\[
, \text{screenDetail = 0 }
\]

\[
, \text{visible = [ ]}
\]

\[
, \text{hidden = [ ]}
\]

\[
, \text{floating = fromList [ ] }
\]

A generalization. When we pass \prop_delete\ to Extrapolate, we instead get the following output (again indented to improve readability):

\[
> \text{check prop_delete}
\]

*** Failed! Falsifiable (after 15 tests):

\[
\text{StackSet}
\]

\[
\text{(Screen (Workspace 0 0 (Just (Stack \{} \text{focus = 'a'}}
\]

\[
\text{\{} \text{up = ''}
\]

\[
\text{\{} \text{down = ''}
\]

\[
\text{\}} \})
\]

\[
, \text{screen = 0}
\]

\[
, \text{screenDetail = 0 }
\]

\[
, \text{visible = [ ]}
\]

\[
, \text{hidden = [ ]}
\]

\[
, \text{floating = fromList [ ] }
\]

Generalization:

\[
\text{StackSet (Screen (Workspace _ _ (Just _))) _ _ _ _ _ _}
\]

Compare the non-generalized counterexample with its generalization. We can see quite clearly which parts are actually related to the fault: what characterizes failing cases is the presence of an optional third argument (of type Maybe Stack) in the argument workspace. Or, as Pike [30] explained “it turns out what matters is having a Just value, which is the stack field that deletion works on!”

We found it easier to exclude the record notation in the output of Extrapolate. In principle, it could be added. Later, in Table 6, we compare the results uniformly avoiding the record notation although some other tools support it.

4.5 Generalizations as property refinements

Whenever Extrapolate finds a generalised condition \( C \) for a property \( P \) to fail and we suspect that the property is incorrect rather than the code under test, we can directly derive from it a candidate variant of that property: \( \neg C \Rightarrow P \). In this way, Extrapolate is also a tool that assists in the refinement of initially conjectured properties, too wide in their scope to be generally true, into more precise variants with scopes defined by conditions.

The actual antecedent introduced in such a refinement may be a simplified equivalent of ‘\( \neg C \)’. Or it may be a different condition, prompted and informed by \( C \), but which the programmer conjectures to be (closer to) the exact necessary and sufficient condition for the property to hold. A programmer using testing without any generalising extrapolation has far more need for such conjectures and has to work harder to find them.

The process of extrapolated checking followed by property refinement may be iterative, terminating when testing finds no counterexample at all. There may be intermediate steps as Extrapolate generalisations are often still approximations — either too general or not general enough. Adding or strengthening an antecedent condition allows us to make progress as further testing reveals new residual counterexamples and their generalisations.

Consider the following property about the \text{words} function defined in the Haskell prelude:

\[
\text{prop_lengthWords :: String -> Bool}
\]

\[
\text{prop_lengthWords s = s /= "" ==} \rightarrow \text{length (words s) == length (filter isSpace s) + 1}
\]

When passed \prop_lengthWords\, Extrapolate reports:

*** Failed! Falsifiable (after 4 tests):

\[
"\n\t"
\]

Generalization:

\[
'\ ':
\]

Conditional Generalization:

\[
c:_ \text{when } c <= '\n\t'
\]

We forgot to account for lists that start (or end) with spaces. The conditional generalization may be puzzling. Here is a clue:

\[
> [c \mid c <$> \text{list}, c <= '\']
\]

\[
"\n\t"
\]

The \( c \) values are drawn from the \text{list} enumeration of test characters. The function \text{words} considers not only ‘\ ’, but also ‘\n’ and ‘\t’ to be spaces.
If we add isSpace to the background, Extrapolate reports:

```haskell
> let chk = check \'withBackground\'
>   [constant "isSpace" isSpace]
> chk prop_lengthWords
...
Conditional Generalization:
c:_ when isSpace c
```

Using this information, we refine our property:

```haskell
prop_lengthWords s =
  s /= "" \&\& not (isSpace (head s))
  \&\& not (isSpace (last s))
  
  length (words s) == length (filter isSpace s) + 1
```

Extrapolate now reports:

```haskell
> chk prop_lengthWords
*** Failed! Falsifiable (after 43 tests):
  "a a"
```

Conditional Generalization:
```
c:' ':' when not (isSpace c)
```

We forgot to account for words separated by more than one space. This becomes even clearer if we add Data.List.isInfixOf and " " to the background and run Extrapolate again on the original prop_lengthWords:

```
Conditional Generalization:
c: when 'isInfixOf' cs
```

We finally reach a correct property of the function words:

```haskell
prop_lengthWords s = noDoubleSpace (" " ++ s ++ " ")
  
  length (words s) == length (filter isSpace s) + 1
  where
    noDoubleSpace s = and [not (isSpace a \&\& isSpace b)
      | (a,b) <- zip s (tail s)]
```

The number of words in a string is given by the number of spaces plus one so long as there are no leading, trailing or double spaces.

### 4.6 Performance Summary

Performance results are summarized in Table 1. For all example applications, Extrapolate takes at most a second to produce unconditional generalizations. For the calculator and XMonad examples, Extrapolate also takes at most a second to produce conditional generalizations. For the sorting and parser applications, Extrapolate takes respectively 5 and 9 seconds to consider conditional generalizations. The Table also includes results for other tools, to be discussed in §6.

Our tool and examples were compiled using ghc -O2 (version 8.2.1) under Linux. The platform was a PC with a 2.2Ghz 4-core processor and 8GB of RAM.

### 5 RELATED WORK

Since the introduction of QuickCheck [10–12], other property-based testing libraries and techniques have been developed. For Haskell, we can cite SmallCheck, Lazy SmallCheck [32, 33] and Feat [15], For Curry, there’s EasyCheck [9], For Erlang, there’s PropEr [29] and QuviQ QuickCheck [1]. For CLEAN, there’s GAST [25].

**Tracing and step-by-step evaluation.** A lot of research has been done on tracing and step-by-step evaluation. In the realm of Haskell, we can note tools such as Freja [27, 28], Hat [7, 8, 37] and Hood [16]. These tools facilitate the process of locating faults in programs. Extrapolate on the other hand does not directly improve this process, but rather gives the programmer improved results to inform it. Except when a generalized counterexample makes it very obvious where the fault is, Extrapolate does not replace tools like Freja, Hat and Hood, but complements them. Claessen et al. [13] explore the combined use of property-based testing and tracing.

**Property discovery and refinement.** QuickSpec [14, 35] and Speculate [5] are tools capable of automatically conjecturing properties given a collection of Haskell functions. Like Extrapolate, these tools rely mainly on testing to achieve their results. Extrapolate does not directly conjecture properties, but its generalized counterexamples can be seen as properties about faults. In Example 1.1, the counterexample `x (x: x: _)` can be seen as the following property: `\x xs \rightarrow not $ prop_sortCount x (x:xs)`

Extrapolate’s generalization aims to find the largest test space in which the negation of the property under test succeeds. As stated in §3, Extrapolate uses Speculate internally.

**Program Synthesis.** Magic Haskeller [19–22] and IGOR II [17, 18, 23] are systems for program synthesis using inductive functional programming techniques [24]. They are able to produce programs based on a limited list of input-output bindings and a set of background functions. Similarly, there are PROGOL [26], FOIL [31] and GOLEM [6] for logic programs. There is potential for the application of these systems and their techniques to generalize counterexamples: based on which test inputs pass or fail, generate a program to describe a set of counterexamples.

**Lazy SmallCheck.** Lazy SmallCheck [32, 33] is a property-based testing tool that uses laziness to prune the search space. Before testing fully defined test values, it tries partially defined test values: if a property fails or passes for a partially defined value, more defined variations of that value need not be tested. As a side-effect of this test strategy, Lazy SmallCheck is able to return partially defined values as counterexamples (see Table 4). These can be read as generalized counterexamples. In §6 we compare results of Lazy SmallCheck to those of Extrapolate.
Table 1: Summary of results for five different applications, testing properties with Extrapolate, SmartCheck and Lazy SmallCheck; #-symbols = #-constants + #-variables; #-constants = number of constants in the reported counterexamples; #-variables = number of variables in the reported counterexamples; generality = how general is the counterexample (○ = least general; ● = most general; × = invalid/loose); runtime = rounded elapsed time; space = peak memory residency. SmartCheck’s result for the faulty XMonad example is presented as reported by Pike [30] and does not include runtime and memory figures.

<table>
<thead>
<tr>
<th>Example &amp; Property</th>
<th>Tool</th>
<th>#-symbols</th>
<th>#-constants</th>
<th>#-variables</th>
<th>generality</th>
<th>Runtime</th>
<th>Memory</th>
</tr>
</thead>
<tbody>
<tr>
<td>Faulty sort (§1)</td>
<td>Ungeneralized</td>
<td>6</td>
<td>6</td>
<td>0</td>
<td>○</td>
<td>&lt; 1s</td>
<td>23MB</td>
</tr>
<tr>
<td>( \forall x \cdot \text{count } x (\text{sort } x) = \text{count } x x )</td>
<td>Lazy SmallCheck</td>
<td>6</td>
<td>6</td>
<td>0</td>
<td>○</td>
<td>&lt; 1s</td>
<td>33MB</td>
</tr>
<tr>
<td></td>
<td>SmartCheck (min)</td>
<td>6</td>
<td>5</td>
<td>1</td>
<td>●</td>
<td>&lt; 1s</td>
<td>22MB</td>
</tr>
<tr>
<td></td>
<td>SmartCheck (median)</td>
<td>12</td>
<td>11</td>
<td>1</td>
<td>○</td>
<td>&lt; 1s</td>
<td>22MB</td>
</tr>
<tr>
<td></td>
<td>Extrapolate</td>
<td>6</td>
<td>2</td>
<td>4</td>
<td>●</td>
<td>&lt; 1s</td>
<td>26MB</td>
</tr>
<tr>
<td></td>
<td>Extrapolate (side)</td>
<td>8</td>
<td>4</td>
<td>4</td>
<td>●</td>
<td>5s</td>
<td>34MB</td>
</tr>
<tr>
<td>Faulty noDiv0 (§4.2)</td>
<td>Ungeneralized</td>
<td>8</td>
<td>8</td>
<td>0</td>
<td>○</td>
<td>&lt; 1s</td>
<td>23MB</td>
</tr>
<tr>
<td>( \forall e \cdot \text{noDiv0 } e )</td>
<td>Lazy SmallCheck</td>
<td>8</td>
<td>7</td>
<td>1</td>
<td>●</td>
<td>&lt; 1s</td>
<td>33MB</td>
</tr>
<tr>
<td></td>
<td>SmartCheck (min/median)</td>
<td>7</td>
<td>6</td>
<td>1</td>
<td>×</td>
<td>&lt; 1s</td>
<td>22MB</td>
</tr>
<tr>
<td></td>
<td>Extrapolate</td>
<td>8</td>
<td>7</td>
<td>1</td>
<td>○</td>
<td>&lt; 1s</td>
<td>26MB</td>
</tr>
<tr>
<td></td>
<td>Extrapolate (side)</td>
<td>10</td>
<td>8</td>
<td>2</td>
<td>●</td>
<td>&lt; 1s</td>
<td>28MB</td>
</tr>
<tr>
<td>Faulty parser (§4.3)</td>
<td>Ungeneralized</td>
<td>17</td>
<td>17</td>
<td>0</td>
<td>○</td>
<td>&lt; 1s</td>
<td>25MB</td>
</tr>
<tr>
<td>( \forall e \cdot \text{read'} (\text{show'} e) = e )</td>
<td>Lazy SmallCheck</td>
<td>17</td>
<td>17</td>
<td>0</td>
<td>○</td>
<td>&lt; 1s</td>
<td>36MB</td>
</tr>
<tr>
<td></td>
<td>SmartCheck (min)</td>
<td>17</td>
<td>17</td>
<td>0</td>
<td>○</td>
<td>&lt; 1s</td>
<td>23MB</td>
</tr>
<tr>
<td></td>
<td>SmartCheck (median)</td>
<td>27</td>
<td>27</td>
<td>0</td>
<td>○</td>
<td>&lt; 1s</td>
<td>23MB</td>
</tr>
<tr>
<td></td>
<td>Extrapolate</td>
<td>14</td>
<td>7</td>
<td>7</td>
<td>●</td>
<td>&lt; 1s</td>
<td>27MB</td>
</tr>
<tr>
<td></td>
<td>Extrapolate (side)</td>
<td>16</td>
<td>7</td>
<td>9</td>
<td>●</td>
<td>9s</td>
<td>35MB</td>
</tr>
<tr>
<td>Faulty XMonad (§4.4)</td>
<td>Ungeneralized</td>
<td>16</td>
<td>16</td>
<td>0</td>
<td>○</td>
<td>&lt; 1s</td>
<td>28MB</td>
</tr>
<tr>
<td>prop_delete</td>
<td>SmartCheck</td>
<td>13</td>
<td>9</td>
<td>4</td>
<td>●</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td></td>
<td>Extrapolate</td>
<td>12</td>
<td>4</td>
<td>8</td>
<td>●</td>
<td>&lt; 1s</td>
<td>30MB</td>
</tr>
<tr>
<td></td>
<td>Extrapolate (side)</td>
<td>15</td>
<td>7</td>
<td>8</td>
<td>●</td>
<td>&lt; 1s</td>
<td>31MB</td>
</tr>
</tbody>
</table>

Table 2: Extrapolate contrasted with Lazy SmallCheck and SmartCheck: ● = yes; ○ = no.

Table 2 compares in summary three tools for Haskell able to report generalized counterexamples: Lazy SmallCheck, SmartCheck and Extrapolate. The key contribution of Extrapolate is allowing for repeated variables and side conditions in generalized counterexamples.
Extrapolate: generalizing counterexamples of functional test properties

We say that a generalization is maximal in the background, Extrapolate reports a generalization – maximal in the sense that it includes a larger set of failing test cases. With the function count in the background, Extrapolate reports a maximal generalization – there is no more general description of the failing cases.

The reported runtime of five seconds needed to reach the condition involving count is for Extrapolate with a maximum explored condition size of 6. With the default settings, Extrapolate reports the condition involving elem in two seconds (§1).

Faulty sort (§1). See Table 3. The counterexample reported by Extrapolate has the same number of symbols as the one reported by Lazy SmallCheck. Lazy SmallCheck is not able to report a generalization as the property being tested is not lazy. Extrapolate is able to generalize the tail and the initial elements of the list whereas SmartCheck is only able to generalize the tail. With the function count in the background, Extrapolate reports a maximal generalization – there is no more general description of the failing cases.

The reported runtime of five seconds needed to reach the condition involving count is for Extrapolate with a maximum explored condition size of 6. With the default settings, Extrapolate reports the condition involving elem in two seconds (§1).

Faulty parser (§4.3). See Table 5. Neither Lazy SmallCheck nor SmartCheck is able to report a generalization. Extrapolate is able to report both an unconditional generalization and to improve it in a further conditional generalization. Extrapolate reports counterexamples that are smaller than those reported by other tools.

Faulty XMonad (§4.4). See Table 6. SmartCheck and Extrapolate give counterexamples of almost the same number of symbols, but the Extrapolate counterexample has fewer constant constructors and more variables. SmartCheck always assigns variable names whereas Extrapolate uses "_" for unreported variables, making it more immediately apparent where component values do not matter. Extrapolate’s conditional and unconditional generalizations are equivalent.

\[\text{let } x0 = \text{Div (C 0) (Add (C 0) (C 0))} \]

\[\text{let } x0 = \text{Div x0 (Add (C (-5)) (C 5))} \]

\[\text{let } x0 = \text{Div (C 0) (Add (C 0) (C 0))} \]

\[\text{let } x0 = \text{Div e1 (Add (C 0) (C 0))} \]

\[\text{when noDiv0 e1} \]

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\[\text{let } x0 = \text{Div (C 0) (Add (C 0) (C 0))} \]

\[\text{let } x0 = \text{Div x0 (Add (C (-5)) (C 5))} \]

\[\text{let } x0 = \text{Div (C 0) (Add (C 0) (C 0))} \]

\[\text{let } x0 = \text{Div e1 (Add (C 0) (C 0))} \]

\[\text{when noDiv0 e1} \]

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Faulty XMonad (§4.4). See Table 6. SmartCheck and Extrapolate give counterexamples of almost the same number of symbols, but the Extrapolate counterexample has fewer constant constructors and more variables. SmartCheck always assigns variable names whereas Extrapolate uses "_" for unreported variables, making it more immediately apparent where component values do not matter. Extrapolate’s conditional and unconditional generalizations are equivalent.

\[\text{let } x0 = \text{Div (C 0) (Add (C 0) (C 0))} \]

\[\text{let } x0 = \text{Div x0 (Add (C (-5)) (C 5))} \]

\[\text{let } x0 = \text{Div (C 0) (Add (C 0) (C 0))} \]

\[\text{let } x0 = \text{Div e1 (Add (C 0) (C 0))} \]

\[\text{when noDiv0 e1} \]
Table 5: Counterexamples for the parser property (§4.3) reported by Extrapolate, SmartCheck and Lazy SmallCheck.

<table>
<thead>
<tr>
<th>Tool</th>
<th>Counterexample</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ungeneralized (full record syntax)</td>
<td>Prog {modules = [], funcs = [Func {fnName = Var &quot;a&quot;, args = [And (Int 0) (Bool False)], stmts = []}]</td>
</tr>
<tr>
<td>Ungeneralized (simplified)</td>
<td>Prog [] [Func (Var &quot;a&quot;) [And (Int 0) (Bool False)] []]</td>
</tr>
<tr>
<td>Lazy SmallCheck</td>
<td>Prog [] [Func (Var &quot;a&quot;) [And (Int 0) (Bool False)] []]</td>
</tr>
<tr>
<td>SmartCheck</td>
<td>Prog [] [Func (Var &quot;U&quot;) [] [Return (And (Bool False) (Int 0))]])</td>
</tr>
<tr>
<td>Extrapolate</td>
<td>Prog _ (Func _ (And Int _) (Bool _::: _:::))</td>
</tr>
<tr>
<td>Extrapolate (with side condition)</td>
<td>Prog _ (Func _ (And e f::: _:::)) when e /= f</td>
</tr>
</tbody>
</table>

Table 6: Counterexamples for prop_delete from XMonad (§4.4) reported by Extrapolate and SmartCheck. The SmartCheck counterexample is the one reported by Pike [30] in the original SmartCheck paper.

<table>
<thead>
<tr>
<th>Tool</th>
<th>Counterexample</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ungeneralized</td>
<td>StackSet (Screen (Workspace 0 0 (Just (Stack 'a' &quot;&quot; &quot;&quot;))) 0 0) [] (fromList [])</td>
</tr>
<tr>
<td>SmartCheck</td>
<td>StackSet (Screen (Workspace x0 (-1) (Just x1)) 1 1) x2 x3 (fromList [])</td>
</tr>
<tr>
<td>Extrapolate</td>
<td>StackSet (Screen (Workspace _ _ (Just _)) _ _) _ _</td>
</tr>
<tr>
<td>Extrapolate (side)</td>
<td>StackSet (Screen (Workspace _ _ ms) _ _) _ _ when Nothing /= ms</td>
</tr>
</tbody>
</table>

7 CONCLUSIONS AND FUTURE WORK

Finally, we note some conclusions and avenues for future work.

7.1 Conclusions

In summary, we have presented a tool that is able to generalize counterexamples of functional test properties. As set out in §2 and §3, Extrapolate enumerates and tests generalizations, reporting the ones that fail all tests. We have demonstrated in §4 Extrapolate’s applicability to a range of examples. After reviewing related work in §5, we have compared in §6 Extrapolate results with those reported by other tools.

Value of reported laws. The conjectured generalizations reported by Extrapolate are surprisingly accurate in practice, despite their inherent uncertainty in principle. They can provide helpful insights into the source of faults. Allowing repeated variables and side-conditions makes possible the discovery of generalizations previously unreachable by other tools that provide similar functionality, such as Lazy SmallCheck [32] and SmartCheck [30].

Ease of use. Extrapolate requires no more programming effort than a regular property-based testing tool such as QuickCheck [11] or SmallCheck [33]. If only standard Haskell datatypes are involved, no extra Listable or Generalizable instances are needed. If user-defined data types can be freely enumerated without a constraining data invariant, instances can be automatically derived.

7.2 Future Work

We note a number of avenues for further investigation that could lead to improved versions of Extrapolate or similar tools.

Type-by-type generalization. Although sufficiently fast for the examples we have tried, the current algorithm to find counterexamples is very naive. It considers generalizations replacing subexpressions of several different types by variables, all in one step. We believe runtime could be reduced by switching to an iterative process where generalization happens one type at a time. First at outer types, then at inner types.

First generalizing with one variable per type. An interesting observation used in our previous work [5] and the work of Smallbone et al. [35] could be used to speed-up Extrapolate. For a property with several variables per type to be true, its one-variable-per-type instance should be true as well, for example:

∀ x y z. (x + y) + z = x + (y + z) ⇒ ∀ x. (x + x) + x = x + (x + x)

The same is true for generalized counterexamples: if lists of the form x:y:xs always falsify a property, then lists of the form x:x:xs should as well. Based on this observation, we can test one-variable-per-type generalizations first, potentially reducing the search space.

Generalizing from several counterexamples. The current version of Extrapolate bases its generalizations on a single counterexample. As mentioned in §5, it may be possible to use techniques from inductive functional programming [24] to base its generalizations on several counterexamples. This might reduce the time needed to produce generalizations or increase their accuracy.

Multiple generalizations. Reported generalizations are derived from initial counterexamples. After finding a generalization, Extrapolate could search for other counterexamples and report additional generalizations. These might be of additional help in finding the source of faults.
Extrapolate: generalizing counterexamples of functional test properties

Parallelism. As a way to improve performance, particularly when dealing with costly test functions, we could parallelize the testing of different enumerated generalizations among multiple processors.

Automatically include functions occurring in properties in the background. In several examples (§4), we provided functions present in the property as background functions for side conditions, improving generalized counterexamples. This could be done automatically to improve out-of-the-box results.

Alternative configuration parameters. Future versions of Extrapolate could offer improved control of configuration parameters. As briefly mentioned in §2, the size limit on side conditions is a very fragile parameter: incrementing it may greatly increase runtime. To make it easier to configure, future versions of Extrapolate could offer a time-limit parameter for when exploring conditions.

Custom generic derivation hierarchy. The improvement of counterexamples in Extrapolate by generalization can be compared with the improvement of counterexamples from QuickCheck by shrinking. In QuickCheck, though there are some proposed default generic derivation rules for shrinking, the shrinking methods are also exposed and custom shrinking functions can be declared explicitly. In Extrapolate, the background is a bit like that: there are defaults, and options to declare more. But the method of defining and searching the hierarchy of generalizations is a fixed default. Allowing it to be overridden could be one way to solve the problem of generalizations when algebraic datatype values cannot be freely generated as there are invariant constraints.

AVAILABILITY
Extrapolate is freely available with a BSD3-style license from:

- https://hackage.haskell.org/package/extrapolate
- https://github.com/rudymatela/extrapolate

This paper describes Extrapolate version 0.3.0.

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